

MIMO channels: optimizing throughput and reducing outage by increasing multiplexing gain

Oboyerulu Agboje, Nsikan Nkordeh, Uzairue Stanley Idiake, Ololade Oladoyin,
Kennedy Okokpujie, Ibinabo Bob-Manuel
Covenant University, Nigeria

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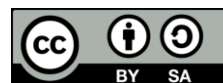
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ABSTRACT

The two main aims of deploying multiple input multiple out (MIMO) are to achieve spatial diversity (improves channel reliability) and spatial multiplexing (increase data throughput). Achieving both in a given system is impossible for now, and a trade-off has to be reached as they may be conflicting objectives. The basic concept of multiplexing: divide (multiplex) transmit a data stream several branches and transmit via several (independent) channels. In this paper, we focused mainly on achieving spatial multiplexing by modeling the channel using the diagonal Bell Labs space time scheme (D-BLAST) and the vertical Bell Labs space time architecture (V-BLAST) Matlab simulations results were also given to further compare the advantages of spatial multiplexing.

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Corresponding Author:

Uzairue Stanley Idiake,

Covenant University, Nigeria.

Email: Stanley.uzairue@covenantuniversity.edu.ng, uzairue.stanley@gmail.com

1. INTRODUCTION

The need for and data rates and a high quality of service (QoS). Over the years, the ubiquity offered by wireless communication has made it the more preferred means over wired; hence, there has been an increase in research on how to improve the modulation schemes used over the air interface. Multiple input multiple output (MIMO) offers desirable properties that meet most of the requirement stated above. By using multiple output multiple input (MIMO) systems, diversity gain mitigates fading, increases coverage and improves QoS. Multiplexing gain increases capacity and spectral efficiency with no additional power or bandwidth expenditure [1]. The core idea under the MIMO systems is the ability to turn multi-path propagation, which is typically an obstacle in conventional wireless communication, into a benefit for users [2]. With MIMO, the capacity of a communication system increases linearly with the number of antennas, thereby achieving an increase in spectral efficiency, without requiring more resources in terms of bandwidth and power [3-5].

From Figure 1 shows that MIMO technology has two main objectives which it aims to achieve: high spatial multiplexing gain and high spatial diversity. To attain spatial multiplexing, the system is made to carry multiple data stream over one frequency, simultaneously-form multiple independent links (on same channel) between transmitter and receiver to communicate at higher data rates. In low SNR environment, spatial diversity techniques are applied to mitigate fading and the performance gain is typically expressed as diversity gain (in dB) [6]; for higher SNR facilitates the use of spatial multiplexing (SM), i.e., the transmission of parallel data streams, and information theoretic capacity in bits per second per Hertz (bits/s/Hz) is the performance measure of choice [7]. Spatial diversity works on the principle of transmission

of structured redundancy improves signal quality, and is able to achieve a high signal to noise ratio (SNR), which translates to high reliability [8].

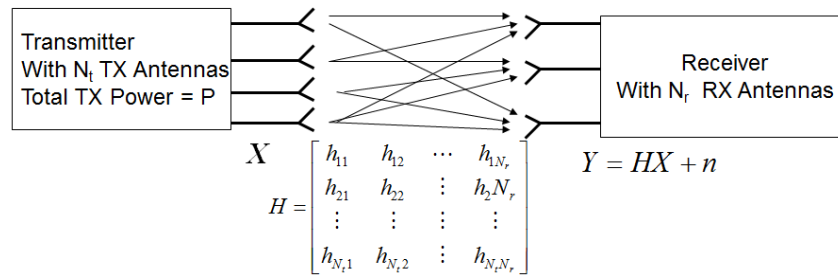


Figure 1. Schematic representation of MIMO [8]

2. RESEARCH METHOD

In today's information and communication technology world, bandwidth and capacity, apart from security, are the most important phenomena for any data related activity. Most IT related processes have a high affinity for large bandwidth, and this places a stringent requirement on the capacity of the communication channel. According to the Shannon-Hartley theorem: the Capacity C of a radio channel is dependent on the bandwidth B and the signal to noise ratio S/N .

$$C = B \log_2(1 + S/N) \quad (1)$$

MIMO technology improve wireless communication channel by two main processes:

- Combating multipath scattering in the communication channel.
- Exploiting, multipath scattering in the communication channel.

In this paper, we limited our work to spatial multiplexing-a mathematical model for Spatial Multiplexing is presented followed by series of simulations using, MATLAB. The multiplexing gain is responsible for MIMO systems offering a linear increase in the achievable data rate. Indeed, in a MIMO channel, multiple independent data streams can be transmitted within the bandwidth of operation and, under suitable channel conditions, these can be separated at the receiver. Furthermore, each data stream experiences at least the same channel quality that would be experienced by a SISO system, effectively enhancing the capacity by a multiplicative factor equal to the number of established streams [8].

To be more specific, we focus on the high SNR regime, and think of a Scheme as a family of codes, one for each SNR level. A scheme is said to have a spatial multiplexing gain r and a diversity advantage d if the rate of the scheme scales like $r \log \text{SNR}$ and the average error probability decays like $1/\text{SNR}$ [8]. H is the $n_t \times n_r$ deterministic matrix. For a MIMO System, the transmit-receive system is represented by:

$$y = Hx + n \quad (2)$$

where:

$$x \in \mathbb{C}^{n_r \times n_t}, y \in \mathbb{C}^{n_r \times n_t}$$

n = white Gaussian noise

the matrix $H \in \mathbb{C}^{n_r \times n_t} \equiv h_{ij}$

To have an insight of spatial multiplexing property, we decouple equation (9) using some common matrix transformation. Using singular value decomposition (SVD), H can be written as:

$$H = U \Lambda V^* \quad (3)$$

where U and V^* are unitary matrices.

Λ is a rectangular matrix whose diagonal elements are non-negative real numbers and whose off-diagonal elements are zero. The diagonal elements $\lambda_1 \geq \lambda_2 \geq \lambda_3 \dots \dots \dots \lambda_{n_{\min}}$.

$$n_{\min} = \min(n_t, n_r)$$

λ_i^2 are the eigenvalues of the matrix HH^* and also of H^*H . The SVD can be rewritten as:

$$H = \sum_{i=1}^{n_{\min}} \lambda_i U_i V_i^* \quad (4)$$

the characteristic of a MIMO system is determined by its multiplexing gain g and its diversity gain d . A Scheme $\{C(SNR)\}$ is said to achieve spatial multiplexing gain g and diversity gain d if the data rate:

$$\lim_{SNR \rightarrow \infty} \frac{R(SNR)}{SNR} = g \quad (5)$$

and average error probability:

$$\lim_{SNR \rightarrow \infty} \frac{\log P_e(SNR)}{\log SNR} = -d \quad (6)$$

The diversity order measures how many statistically independent copies of the same symbol the receiver is able to get in order to reproduce a reliable estimate of the transmitted symbol. Assume the fading coefficient matrix H is known to the receiver, the channel capacity (bps/Hz) of a system with M transmit and N receive antennas is given by:

$$\begin{aligned} C_{\text{Coherent}}(SNR) &= E \left[\log_2 \det \left(I_N + \frac{SNR}{M} HH^* \right) \right] \\ &= E \left[\log_2 \det \left(I_N + \frac{SNR}{M} H^* H \right) \right] \end{aligned} \quad (7)$$

introducing 2 boundaries for limits $K = \min\{M, N\}$ $K' = \max\{M, N\}$. For the case of $M = N$, at high SNR

$$C_{\text{Coherent}} = M \log_2 \frac{SNR}{M} + \sum_{i=1}^M E[\log_2 X_{2i}^2] + O(1) \quad (8)$$

there is a decrease in error probability as we increase the number of antennas, which mean an increase in SNR will result in a decrease in error Probability; we also achieve decrease error probability by deploying higher modulation schemes. A plot of the symbol error rate (SER) to the SNR gives a lot of important and revealing information; the slope of this curve gives us the value of the diversity gain d . The multiplexing gain g gives a measure of how fast spectral efficiency can increase with increase of SNR while keeping the same error rate; the corresponds to the maximum number of independent layers of parallel channel [9], and is limited by:

$$g_{\max} = \min(N_T, N_R) \quad (9)$$

the multiplexing gain of a MIMO systems depends on the type of modulation scheme and the SNR. The probability of error for an $N=M=1$ system using PSK modulation is given by [8] as:

$$P_e(SNR) \approx \frac{1}{4} SNR^{-1} \quad (10)$$

for a system with two receivers transmitting the same signal, the error probability is given as:

$$P_e(SNR) \approx \frac{3}{16} SNR^{-2} \quad (11)$$

it can be shown from spectral analysis that the data rate (multiplexing gain) of a SISO system using 4-PAM modulation scheme (2bit/s/Hz) can be improved without a depreciation in error rate (keeping the same error) by switch to an 8-PAM modulations scheme (3bits/s/Hz) is the SNR is increased by 6 dB; a further 6 dB increase in the SNR with the use of 64-QAM modulation scheme (3 bits/s/Hz) will increase

the multiplexing gain while keeping the error rate the same as when 4-PAM modulation scheme was used [9].

Ergodic capacity represents the maximum achievable throughput averaged across all fading conditions. Multi-path fading properties of a channel change over relatively longer timescales (compared to receiver noise) and the fading state stays fairly constant over an interval called the channel coherence interval. Reliable communication at rates arbitrarily close to the ergodic capacity requires averaging across many independent realizations of the channel gains over time. Since the channel capacity increases linearly with $\log \text{SNR}$, in order to achieve a certain fraction of the capacity at high SNR, we should consider schemes that support a data rate which also increases with SNR. Here, we think of a *scheme* as a family of codes $fC(\text{SNR})$ of block length T , one at each SNR level. Let $R(\text{SNR})$ (bits/symbol) be the rate of the code $C(\text{SNR})$. We say that a scheme achieves a *spatial multiplexing gain* of r if the supported data rate [10, 11].

$$R(\text{SNR}) \approx r \log \text{SNR} \quad \text{bps/Hz} \quad (12)$$

Spatial multiplexing gain can also be thought as the data rate normalized with respect to the SNR level. A common way to characterize the performance of a communication scheme is to compute the error probability as a function of SNR for a fixed data rate. A plot of error probability against normalized SNR was proposed by Forney in order to compare these schemes fairly.

$$\text{SNR}_{\text{norm}} \triangleq \frac{\text{SNR}}{C^{-1}(R)} \quad (13)$$

where $C(\text{SNR})$ is the capacity of the channel as a function of SNR. SNR_{norm} measures how far the SNR is above the minimal required to support the target data rate. Another way to characterize the performance of a MIMO system is to plot error probability against normalized data rate, a fixed SNR .

$$R_{\text{norm}} \triangleq \frac{R}{C(\text{SNR})} \quad (14)$$

Notice that at high SNR, the capacity of the multiple antenna channel is $C(\text{SNR}) \approx K \log \text{SNR}$; hence the spatial multiplexing gain.

$$g = \frac{R}{\log \text{SNR}} \simeq k R_{\text{norm}} \quad (15)$$

Two of the ways multiplexing gain can be achieved are: V-BLAST Vertical Bell Labs Space Time architecture and D-BLAST, Diagonal Bell Lab Space Time Scheme [10, 11].

2.1. V-BLAST

The vertical Bell Labs space time architecture (V-BLAST) attempts to maximize system throughput at the cost of averaging across fewer fading coefficients. Under V-BLAST, the message is split across two coded streams. \mathbf{X} is represented as follows:

$$\begin{bmatrix} x_1(1) & x_1(2) & x_1(3) & \dots & \dots \\ x_2(1) & x_2(2) & x_2(3) & \dots & \dots \end{bmatrix} \quad (16)$$

where $\{x_1[i]\}_{i=1}^N$ and $\{x_2[i]\}_{i=1}^N$ are components of two independent achieving code with rates R_1 and R_2 respectively. For both streams, the transmit antenna remains fixed and coding is performed only across time. Unlike in the Alamouti scheme where two information symbols appear orthogonal at the receiver due to the special construction of \mathbf{X} . In V-BLAST, such a construction is not used and the receiver estimates $x_1[i]$ and $x_2[i]$ from its received signal using signal processing techniques. For a 2×2 MIMO channel with V-BLAST decomposes the analysis into two SISO parallel sub-channels having complex gains $P_1(H)$ and $P_2(H)$ respectively. The outage probability for this scheme is given as [12-15]:

$$P_{out}^{VBLAST}(SNR, R_1, R_2) = P \left[\bigcup_{i=1}^2 (\log(1 + SNR |p_i(H)|^2) < R_i) \right] \quad (17)$$

2.2. D-BLAST

The diagonal Bell Labs space time scheme (D-BLAST) uses a diagonal structure and performs coding across antennas as well as space. The sequence $X' = \{X'[i]\}_{i=1}^N$ forms one code word belonging to a capacity achieving outer code of rate R . Each codeword is then split into two blocks of symbols X'_A and X'_B , each having $N/2$ symbols [15-18]. The transmitter therefore sends out the following code word X given as:

$$\begin{bmatrix} \dots X'_B[i] & X'_B[i] \dots \\ \dots X'_A[i+1] & X'_A[i+2] \dots \end{bmatrix} \quad (18)$$

in order to initialize the diagonal block structure shown above, one initial block of $N/2$. Symbols i.e. $X'_B[0]$ is set to 0. If one considers the information rate for D-BLAST over a sufficiently large number of codewords, this one-time overhead has negligible effect. The diagonal structure also allows successive interference cancellation; using D-BLAST, the MIMO channel decomposes into two parallel SISO sub-channels having gains $g_1(H)$ and $g_2(H)$ [19]. The outage probability for D-BLAST is given as follows [14-16]:

$$P_{out}^{DBLAST}(SNR, R) = P \left[\sum_{k=1}^2 (\log(1 + SNR |g_k(H)|^2) < R) \right] \quad (19)$$

the joint distribution of $g_1(H)$ and $g_2(H)$ is determined by the receiver architecture. Outage probability is another standard performance criterion denoted by P_{out} and defined as the probability that the instantaneous channel capacity below a specified value, or equivalently, the probability that the output SNR (or SINR) falls below a pre-defined acceptable threshold. Mathematically speaking, the outage probability is the c.d.f. of SNR evaluated at the specified threshold, i.e.

$$\int_0^{\gamma_{th}} p_y(\gamma) d\gamma \quad (20)$$

γ_{th} is the predefined threshold and $p_y(\gamma)$ is the p.d.f of SNR γ .

The average SER, denoted by P_{SER} , is the one that is most revealing about the nature of the system behavior and is generally the most difficult performance criterion to compute. It is defined as the probability that a transmitted data symbol is detected in error at the receiver. The SER is typically modulation/detection scheme dependent, and is directly related to the instantaneous SNR (or SINR for multiuser systems) [20-25].

3. SIMULATION RESULTS

Figure 2 shows the simulation result of the achievable rate transmission to the average SNR of MIMO receivers under fast fading channel condition. From this graph we can see that the SIC has the highest SNR (signal to Noise ratio) while MMSE and Zero-forcing are of this value of SNR. Figure 3 shows how the performance improves with increasing multiplexing gains, where the receiving end uses SISO, quasi-orthogonal, STBC and Orthogonal STB; the BER is best for the receiving system using orthogonal STBC and worst for the SISO. Figure 4 shows the simulation result of the outage probability for 1-stream 4x4 MISO Rayleigh to the average SNR under slow fading condition. That is when the fading rate of the SNR is below normal.

The simulations in Figure 5 and Figure 6 shows that by increasing the antenna system, there is reduction in error rates which helps to improve the availability of the channel: the system using the 4x4 MIMO performs best compared to the systems using 1x4 SIMO, 4x1 MISO and 1x1 SISO, the 1x1 SISO system performs worst. It is interestingly noticed that the 1x4 SIMO system performs better than the 4x1 MISO system. This means in a case where a choice of deploying multiple antenna at the input or output is to be made, it is better to deploy the multiple antenna at the output.

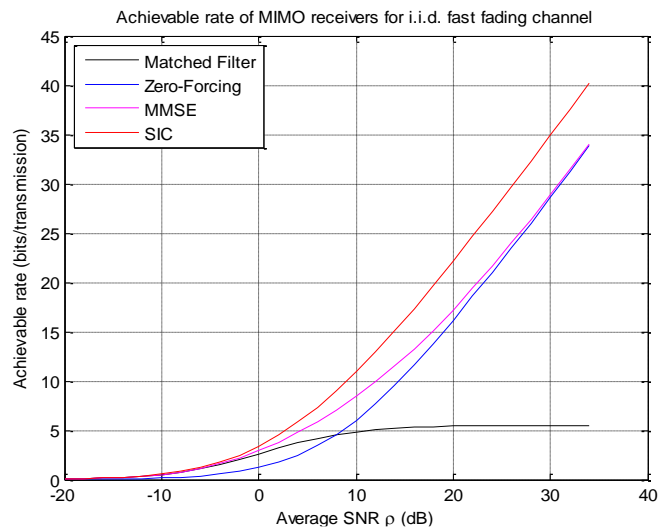


Figure 2. Achievable rate of MIMO receivers for i.i.d fast fading channel

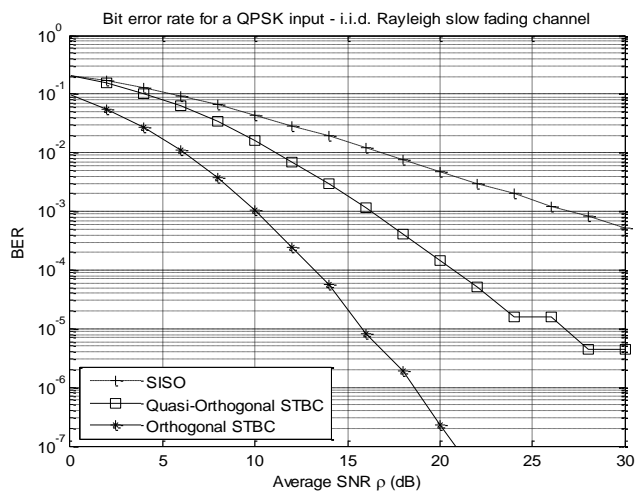


Figure 3. Bit error rate for a QPSK input-i.i.d rayleigh slow fading channel

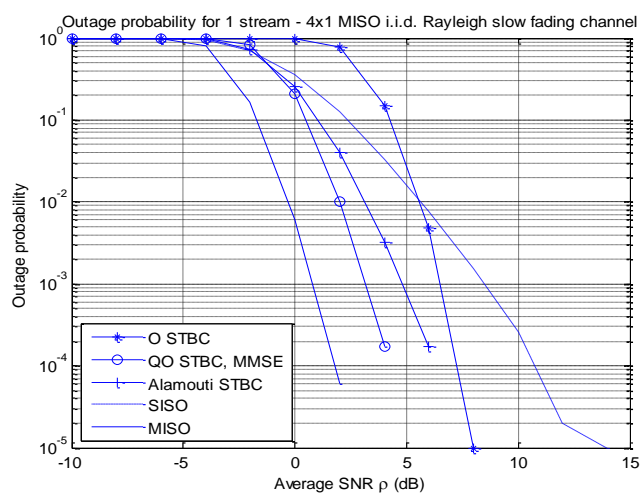


Figure 4. Outage probability for 1 stream-4 x 1 MISO i.i.d rayleigh slow fading channel

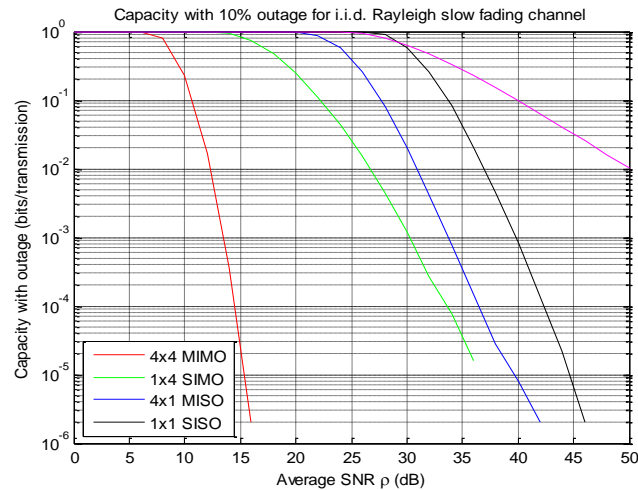


Figure 5. Capacity with 10% outage for i.i.d rayleigh fading channel

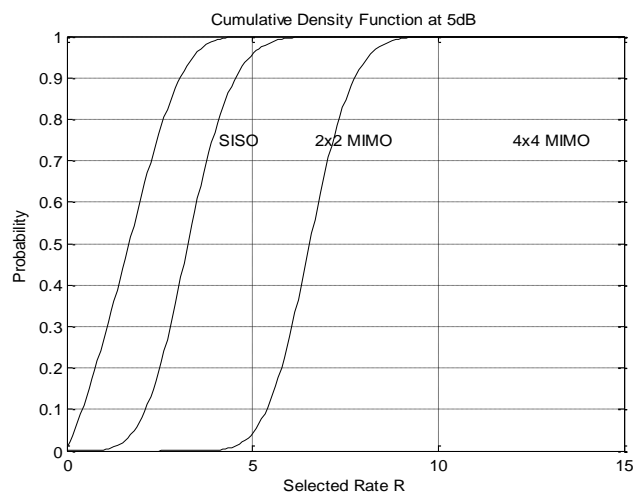


Figure 6. Cumulative density function at 5dB

4. CONCLUSION

Simulations were carried out in Matlab to demonstrate different scenarios of MIMO and also to show how the performance improves with increasing multiplexing gains. The simulation results are shown in the graphs of Figure 2 to Figure 6. The simulations show how outage is reduced by multiplexing. Figure 3 shows how the performance improves with increasing multiplexing gains, where the receiving end uses SISO, Quasi-Orthogonal, STBC and Orthogonal STB; the BER is best for the receiving system using Orthogonal STBC and worst for the SISO. The simulations in Figure 5 shows that by increasing the antenna system, there is reduction in error rates which helps to improve the availability of the channel: the system using the 4x4 MIMO performs best compared to the systems using 1x4 SIMO, 4x1 MISO and 1x1 SISO. The 1x1 SISO system performs worst. It is interestingly noticed that the 1x4 SIMO system performs better than the 4x1 MISO system. This means in a case where a choice of deploying multiple antenna at the input or output is to be made, it is better to deploy the mutiple antenna at the output.

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